Ambiguous Case Worksheet
Solutions

$(15 > 12)$

1. a) $a > b$ - the opposite side is greater than the lonely side.

$\therefore$ One Triangle Possible.

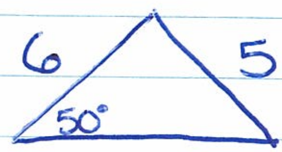
$(6 > 5)$

b) $b > a$ so need to find height

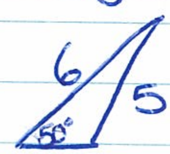
$h = b\sin A = 6 \cdot \sin 50 = 4.5963$

$h < a < b$

$4.5963 < 5 < 6$ $\therefore$ Two triangles are Possible.



and

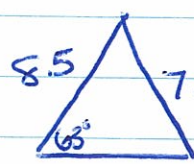


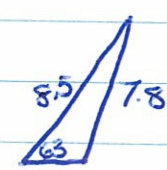
c) $b > a$ so need to find height

$(8.5 > 7.8)$ $h = b\sin A = 8.5 \cdot \sin 63 = 7.5736$

$h < a < b$

$7.5736 < 7.8 < 8.5$ $\therefore$ Two triangles are possible



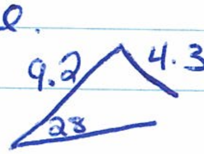


d) $b > a$ so need to find height

$(9.2 > 4.3)$ $h = b\sin A = 9.2\sin 28 = 4.3191$

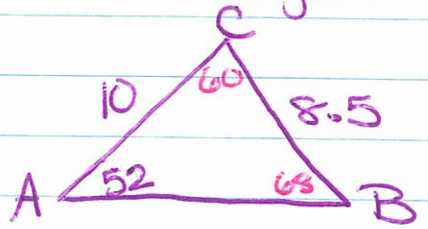
$a < h$ - the opposite side is shorter than the height so it is impossible to form a triangle.

$\therefore$ No Triangle is Possible



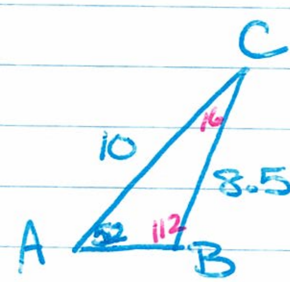
2. $\angle A = 52°$ $a = 8.5$ $b = 10$

Triangle 1



use sine law to find B

$$\frac{\sin B}{10} = \frac{\sin 52}{8.5}$$



$$\sin B = \frac{10 \cdot \sin 52}{8.5}$$

$\sin B = 0.9270714748$

$B = \sin^{-1}(0.9270714748)$

$B = 67.98$

$B = 68°$ → OR $B = 180 - 68 = 112°$

find C:

$180 - 52 - 68 = 60°$

find C:

$180 - 52 - 112 = 16°$

use sine law or Cosine law to find side c.

$$\frac{c}{\sin 60} = \frac{10}{\sin 68}$$

$$c = \frac{10 \cdot \sin 60}{\sin 68}$$

$c = 9.3$

$B = 68°$
$C = 60°$
$c = 9.3\text{cm}$

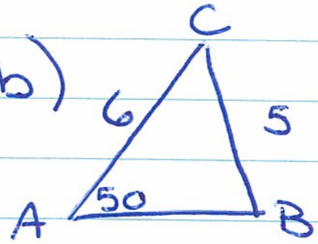
$$\frac{c}{\sin 16} = \frac{8.5}{\sin 52}$$

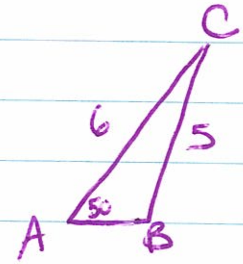
$$c = \frac{8.5 \cdot \sin 16}{\sin 52}$$

$c = 2.973 = 3.0$

$B = 112°$
$C = 16°$
$c = 3.0\text{cm}$

3. (#1b)





$$\frac{\sin B}{6} = \frac{\sin 50}{5}$$

$$\sin B = \frac{6 \cdot \sin 50}{5}$$

$B = 67°$ → OR $180 - 67 = 113°$

$C = 180 - 50 - 67$

$C = 63°$

$$\frac{c}{\sin 63} = \frac{5}{\sin 50}$$

$$c = \frac{5 \cdot \sin 63}{\sin 50} = 5.8$$

$\boxed{c = 5.8\text{ cm}}$

$C = 180 - 50 - 113$

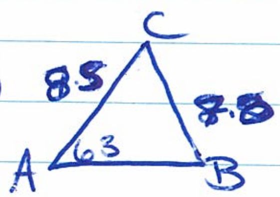
$C = 17°$

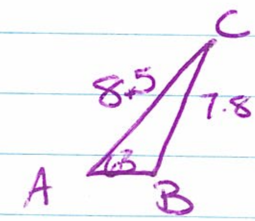
$$\frac{c}{\sin 17} = \frac{5}{\sin 50}$$

$$c = \frac{5 \cdot \sin 17}{\sin 50} = 1.9$$

$\boxed{c = 1.9\text{ cm}}$

(#1c)





$$\frac{\sin B}{8.5} = \frac{\sin 63}{7.8}$$

$B = 76°$ → OR $180 - 76 = 104°$

$C = 180 - 63 - 76 = 41°$

$$\frac{c}{\sin 41} = \frac{7.8}{\sin 63}$$

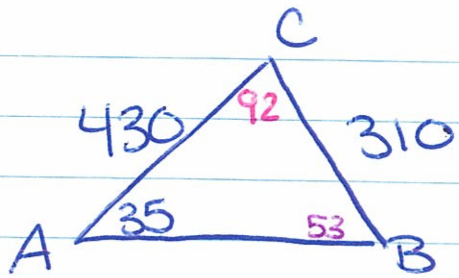
$\boxed{c = 5.7\text{ cm}}$

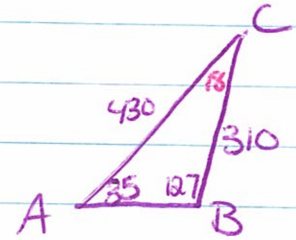
$C = 180 - 63 - 104 = 13°$

$$\frac{c}{\sin 13} = \frac{7.8}{\sin 63}$$

$\boxed{c = 2.0\text{ cm}}$

4.





$$\frac{\sin B}{430} = \frac{\sin 35}{310}$$

$B = 53°$ → **OR** $180 - 53 = 127°$

$C = 180 - 53 - 35 =$
$C = 92°$

$C = 180 - 35 - 127 =$
$C = 18°$

$$\frac{c}{\sin 92} = \frac{310}{\sin 35}$$

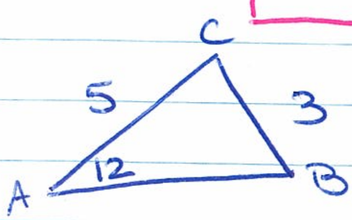
$c = 540\text{ m}$

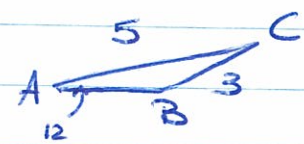
$$\frac{c}{\sin 18} = \frac{310}{\sin 35}$$

$c = 167\text{ m}$

The 3rd side is either
540m or 167m

5.





$$\frac{\sin B}{5} = \frac{\sin 12}{3}$$

$B = 20°$ → **OR** $180 - 20 = 160°$

$C = 180 - 12 - 20 = 148°$

$C = 180 - 12 - 160 = 8°$

$$\frac{c}{\sin 148} = \frac{3}{\sin 12}$$

$c = 7.6\text{ km}$

$$\frac{c}{\sin 8} = \frac{3}{\sin 12}$$

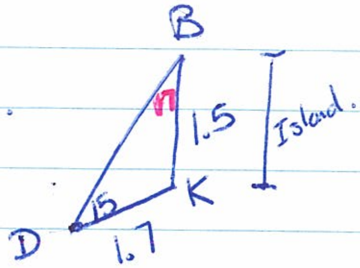
$c = 2.0\text{ km}$

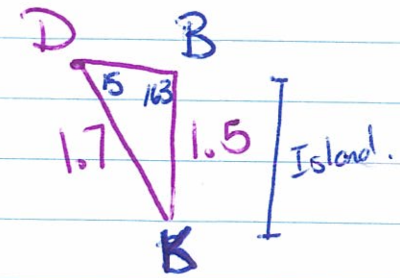
The Raven's Song is either
7.6 km or 2.0 km from the dock.

$h = 1.7 \sin 15 = 0.45$ height < opposite side < lonely side

$\Rightarrow$ Ambiguous Case $2\triangle$'s

6.





$$\frac{\sin B}{1.7} = \frac{\sin 15}{1.5}$$

$B = 17°$ → **OR** $180 - 17 = 163°$

$K = 180 - 15 - 17$
$= 148°$

$K = 180 - 15 - 163$
$= 2°$

$$\frac{k}{\sin 148} = \frac{1.5}{\sin 15}$$

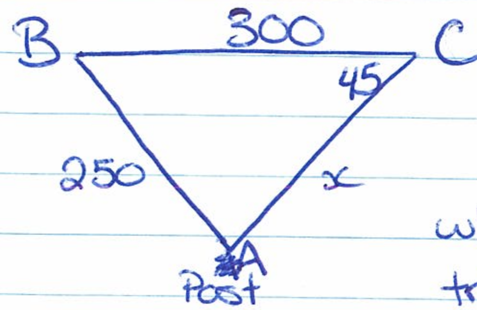
$k = 3.1$ km

$$\frac{k}{\sin 2} = \frac{1.5}{\sin 15}$$

$k = 0.2$ km

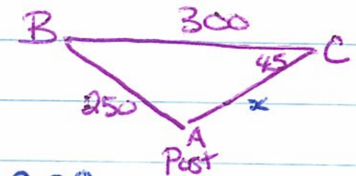
The possible distances for the kayak portion of the race are 3.1 km or 0.2 km. 3.1 km is most likely because 0.2 km is a bit short for kayak race.

#11.

7. 

$h = \text{lonelyside} \cdot \sin 45 = 300 \sin 45$
$= 212$

height < opposite side < lonely side
which means there are 2 possible triangles and 2 different values for x.



$$\frac{\sin A}{300} = \frac{\sin 45}{250}$$

$A = 58°$ → OR $180 - 58 = 122°$

$B = 180 - 45 - 58 = 77°$

$B = 180 - 45 - 122 = 13°$

$$\frac{x}{\sin 77} = \frac{250}{\sin 45}$$

$$\frac{x}{\sin 13} = \frac{250}{\sin 45}$$

$x = 344.4918$
$= 344$ yd

$x = 79.5322$
$x = 80$ yd

He either needs to walk 344yd or 80 yd.